Solution of Previous Paper of GATE

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Q.1 Consider the linear differential equation $\frac{dy}{dx} = xy$. If $y = 2$ at $x = 0$, then the value of y at $x=2$ is given by

(A) e^{-2}
(B) $2e^{-2}$
(C) e^{2}
(D) $2e^{2}$

:-
$\frac{dy}{dx} = xy$, $y = 2$ at $x = 0$

$\frac{dy}{dx} - xy = 0$

$I.F. = e^{-\int x\,dx} = e^{-x^2/2}$

Now
$y \times IF = \int Q \times I.F.\, dx + c$

$y e^{-x^2/2} = 0 + c$

$y = ce^{x^2/2}$

at $x = 0$
$2 = ce^{0} \Rightarrow c = 2$

$\Rightarrow y = 2e^{x^2/2}$

at $x = 2$
$y = 2e^{2} \Rightarrow$ option (d)

Q.2 Which of the following magnetic vector potentials gives rise to a uniform magnetic field $B_0\hat{k}$?

(A) $B_0 z\hat{k}$
(B) $-B_0 x\hat{j}$
(C) $\frac{B_0}{2}\left(-y\hat{i} + x\hat{j}\right)$
(D) $\frac{B_0}{2}\left(y\hat{i} + x\hat{j}\right)$

Sol.:- $\vec{B} = B_0\hat{k}$

we know that

$\vec{B} = \vec{\nabla}\times\vec{A}$

Now, we ~~ckec~~ check options

(A) $B_0 z\hat{k} = \vec{A}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & z \end{vmatrix} = \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(0-0) = 0 \neq \vec{B} \times$$

(B) $\vec{A} = -B_0 x\hat{j}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -B_0x & 0 \end{vmatrix} = \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}(-B_0-0) = -B_0\hat{k} \neq \vec{B} \times$$

(C) $\vec{A} = B_0/2(-y\hat{i}+x\hat{j})$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\frac{B_0}{2}y & \frac{B_0}{2}x & 0 \end{vmatrix} = \hat{i}(0-0)-\hat{j}(0-0)+\hat{k}\left(\frac{B_0}{2}+\frac{B_0}{2}\right) = B_0\hat{k} = \vec{B} \checkmark$$

Q.3 The molecule $^{17}O_2$ is

(A) Raman active but not NMR (nuclear magnetic resonance) active.
(B) Infrared active and Raman active but not NMR active.
(C) Raman active and NMR active.
(D) Only NMR active.

ANS-(C)

Q.4 There are four electrons in the 3d shell of an isolated atom. The total magnetic moment of the atom in units of Bohr magneton is ________.

Sol:- four e^- in 3d shell

3d	↿	↿	↿	↿	
$l=2$	-2	-1	0	$+1$	$+2$

$m_l = -2-1+0+1 = -2$

$m_s = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = 2$

Now

magnetic moment $= -(m_l+m_s)\mu_B$

$= -(-2+2)\mu_B$

$\boxed{\mu = 0}$

Q.5 Which of the following transitions is NOT allowed in the case of an atom, according to the electric dipole radiation selection rule?

(A) 2s-1s (B) 2p-1s (C) 2p-2s (D) 3d-2p

We know that electric dipole radiation selection rule

$$\Delta l = +1$$

So, 2s-1s is not allowed

$\Rightarrow$ option (a)

Q.6 In the SU(3) quark model, the triplet of mesons (π^+, π^0, π^-) has

(A) Isospin = 0 , Strangeness = 0
(B) Isospin = 1 , Strangeness = 0
(C) Isospin = 1/2, Strangeness = +1
(D) Isospin = 1/2, Strangeness = -1

The triplet of mesons (π^+, π^0, π^-)

Isospin – $2I+1 = 3$

$$\boxed{I = 1}$$

we know that

$$-I \leq I_3 \leq I$$

$$I_3(\pi^+) = 1 \quad , I_3(\pi^0) = 0 \ , \ I_3(\pi^-) = -1$$

Now, we know that

$$Q = I_3 + \frac{B+S}{2}$$

$$+1 = 1 + \frac{0+S}{2}$$

$$\boxed{S = 0}$$

$\Rightarrow$ option (b)

Q.7 The magnitude of the magnetic dipole moment associated with a square shaped loop carrying a steady current I is m. If this loop is changed to a circular shape with the same current I passing through it, the magnetic dipole moment becomes $\frac{pm}{\pi}$. The value of p is ______.

:- let side of is a and radius of circle r

$$m = Ia^2$$

Now

$$4a = 2\pi r$$

$$r = \frac{2a}{\pi}$$

$$m_2 = IA$$

$$= I \times \pi r^2$$

$$= I \times \pi \frac{4a^2}{\pi^2} = \frac{4}{\pi} Ia^2$$

$$m_2 = \frac{4m}{\pi} \quad \Longrightarrow \boxed{P = 4}$$

Q.8 The total power emitted by a spherical black body of radius R at a temperature T is P_1. Let P_2 be the total power emitted by another spherical black body of radius $R/2$ kept at temperature $2T$. The ratio, P_1/P_2 is _______. (Give your answer upto two decimal places)

$$P_1 = \frac{E_1}{t}$$
$$P_2 = E_2/t \quad \Rightarrow \quad \frac{P_1}{P_2} = \frac{E_1}{E_2}$$
$$E_1 \propto T_1^4 R_1^2$$
$$E_2 \propto T_2^4 R_2^2$$
$$T_2 = 2T_1 \quad , \quad R_2 = \frac{R_1}{2}$$
$$\frac{E_1}{E_2} = \frac{T^4 R^2}{(2T)^4 (R/2)^2} = \frac{1}{4} = 0.25$$
$$\boxed{P_1/P_2 = 0.25}$$

Q.9 The entropy S of a system of N spins, which may align either in the upward or in the downward direction, is given by $S = -k_B N\left[p \ln p + (1-p)\ln(1-p)\right]$. Here k_B is the Boltzmann constant. The probability of alignment in the upward direction is p. The value of p, at which the entropy is maximum, is _______. (Give your answer upto one decimal place)

$$S = -k_B N\left[p \ln p + (1-p)\ln(1-p)\right]$$

for max. entropy

$$\frac{dS}{dp} = 0$$
$$-k_B N\left[\ln p + \frac{p}{p} + \frac{-(1-p)}{(1-p)} + (-1)\ln(1-p)\right] = 0$$
$$\left[\ln p + \cancel{1}0 - \ln(1-p)\right] = 0$$
$$\ln\left(\frac{p}{1-p}\right) = 0$$
$$\Rightarrow \frac{p}{1-p} = e^0$$
$$\frac{p}{1-p} = 1$$
$$p = 1-p$$
$$\boxed{p = 1/2}$$

Q.10 For a system at constant temperature and volume, which of the following statements is correct at equilibrium?

(A) The Helmholtz free energy attains a local minimum.
(B) The Helmholtz free energy attains a local maximum.
(C) The Gibbs free energy attains a local minimum.
(D) The Gibbs free energy attains a local maximum.

$T = \text{const} \Rightarrow dT = 0$

$V = \text{const} \Rightarrow dV = 0$

we know that

$dF = -SdT - PdV$

$dF = 0$

$dG = VdP - SdT$

$dG = VdP$

$\Longrightarrow$ option ⓐ

Q.11 N atoms of an ideal gas are enclosed in a container of volume V. The volume of the container is changed to $4V$, while keeping the total energy constant. The change in the entropy of the gas, in units of $Nk_B \ln 2$, is _______ , where k_B is the Boltzmann constant.

this is case of free expansions

$$\Delta S = Nk_B \int_V^{4V} \frac{1}{V} dV$$

$$= Nk_B [\ln V]_V^{4V} \Rightarrow \Delta S = Nk_B \ln\left(\frac{4V}{V}\right)$$

$$\Delta S = Nk_B \ln(4)$$

$$\Delta S = 2Nk_B \ln(2)$$

$\Longrightarrow$ ans. is ②

Q.12 Which of the following is an analytic function of z everywhere in the complex plane?

(A) z^2 (B) $(z^*)^2$ (C) $|z|^2$ (D) $\sqrt{z}$

sol :- For analytic function check options

(a) $f(z) = z^2$

$f(z) = (x+iy)^2$

$= (x^2 - y^2 + 2xiy)$

$f(z) = (x^2 - y^2) + i(2xy)$

$u(x,y) = x^2 - y^2$

$v(x,y) = 2xy$

Now, from C-R equations

$$\frac{du}{dx} = \frac{dv}{dy}$$

$2x = 2x$ ✓

$$\frac{du}{dy} = -\frac{dv}{dx}$$

$-2y = -2y$ ✓

$\Rightarrow$ option ⓐ

Q.13 In a Young's double slit experiment using light, the apparatus has two slits of unequal widths. When only slit-1 is open, the maximum observed intensity on the screen is $4I_0$. When only slit-2 is open, the maximum observed intensity is I_0. When both the slits are open, an interference pattern appears on the screen. The ratio of the intensity of the principal maximum to that of the nearest minimum is ________.

$I_1 = 4I_0$

$I_2 = I_0$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \frac{(2\sqrt{I_0}+\sqrt{I_0})^2}{(2\sqrt{I_0}-\sqrt{I_0})^2} = \frac{9I_0}{I_0}$$

$$\frac{I_{max}}{I_{min}} = 9$$

Q.14 Consider a metal which obeys the Sommerfeld model exactly. If E_F is the Fermi energy of the metal at $T = 0$ K and R_H is its Hall coefficient, which of the following statements is correct?

(A) $R_H \propto E_F^{3/2}$ (B) $R_H \propto E_F^{2/3}$ (C) $R_H \propto E_F^{-3/2}$ (D) R_H is independent of E_F.

ANS-(C)

Q.15 A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?

(A) The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.
(B The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.
(C) Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.
(D) Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes.

ANS-(A)

Q.16 Which of the following operators is Hermitian?

(A) $\frac{d}{dx}$ (B) $\frac{d^2}{dx^2}$ (C) $i\frac{d^2}{dx^2}$ (D) $\frac{d^3}{dx^3}$

sol^n:- for Hermitian operator

$\hat{A}^\dagger = \hat{A}$

check options

(a) $\hat{A} = \frac{d}{dx}$

$\hat{A} = \left(\frac{d}{dx}\right)^\dagger = -\frac{d}{dx} = -\hat{A} \neq \hat{A}$ ✗

(b) $\hat{A} = \frac{d^2}{dx^2}$

$\hat{A}^\dagger = \left(\frac{d^2}{dx^2}\right)^\dagger = \left(\left(\frac{d}{dx}\right)^\dagger\right)^2 = \left(-\frac{d}{dx}\right)^2 = \frac{d^2}{dx^2}$

$\hat{A}^\dagger = \hat{A}$ ✓

$\Rightarrow$ option (b)

Q.17 The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of m_0c, where c is the speed of light in vacuum, is _______. (Give your answer upto two decimal places)

:-

K.E. = rest mass energy = m_0c^2

we know that

$$E^2 = p^2c^2 + m_0^2c^4$$

$$(K.E. + m_0c^2)^2 = c^2(p^2 + m_0^2c^2)$$

$$\Rightarrow (2m_0c^2)^2 = c^2(p^2 + m_0^2c^2)$$

$$\Rightarrow 4m_0^2c^4 = c^2(p^2 + m_0^2c^2)$$

$$\Rightarrow p^2 + m_0^2c^2 = 4m_0^2c^2$$

$$\Rightarrow p^2 = 3m_0^2c^2$$

$$\Rightarrow p = \sqrt{3}\, m_0c$$

$\Rightarrow$ Answer is $\boxed{\sqrt{3} = 1.73}$

Q.18 The number density of electrons in the conduction band of a semiconductor at a given temperature is 2×10^{19} m^{-3}. Upon lightly doping this semiconductor with donor impurities, the number density of conduction electrons at the same temperature becomes 4×10^{20} m^{-3}. The ratio of majority to minority charge carrier concentration is ________.

sol^n :-

for n-type

$$n \cdot p = n_i^2$$

$$n_i = 2\times10^{19}, \quad n = 4\times10^{20}$$

$$p = \frac{(2\times10^{19})^2}{4\times10^{20}} = 10^{18}$$

$$\frac{\text{Majority c.c.}}{\text{Minority c.c.}} = \frac{4\times10^{20}}{10^{18}} = 400$$

Q.19 Two blocks are connected by a spring of spring constant k. One block has mass m and the other block has mass $2m$. If the ratio $k/m = 4$ s^{-2}, the angular frequency of vibration ω of the two block spring system in s^{-1} is _______. (Give your answer upto two decimal places)

:-

for electron bo blocks reduce mass

$$\mu = \frac{m\times2m}{m+2m} = \frac{2}{3}m$$

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{2/3\,m}} = \sqrt{\frac{3}{2}}\sqrt{\frac{k}{m}}$$

$$\omega = \frac{\sqrt{3}}{\sqrt{2}}\times2 = \sqrt{6} = 2.44$$

Q.20 A particle moving under the influence of a central force $\vec{F}(\vec{r}) = -k\vec{r}$ (where $\vec{r}$ is the position vector of the particle and k is a positive constant) has non-zero angular momentum. Which of the following curves is a possible orbit for this particle?

(A) A straight line segment passing through the origin.
(B) An ellipse with its center at the origin.
(C) An ellipse with one of the foci at the origin.
(D) A parabola with its vertex at the origin.

ANS-(B)

Q.21 Consider the reaction ${}^{54}_{25}Mn + e^- \rightarrow {}^{54}_{24}Cr + X$. The particle X is

(A) γ (B) ν_e (C) n (D) π^0

ANS-(B)

Q.22 The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresponding Born approximation is known as the first Born approximation. Such an approximation is valid for

(A) large incident energies and weak scattering potentials.
(B) large incident energies and strong scattering potentials.
(C) small incident energies and weak scattering potentials.
(D) small incident energies and strong scattering potentials.

ANS-(A)

Q.23 Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90^0 and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}$ fm^{-1} then the total scattering cross section in units of fm^2 is _______.

We know that

$$\sigma = \sum_l \frac{4\pi}{k^2}(2l+1)\sin\delta_l$$

$$= \frac{4\pi}{k^2}(2\times 0+1)\sin\delta_0$$

$$= \frac{4\pi}{(\sqrt{2\pi})^2}\sin\frac{\pi}{2}$$

$$= \frac{4\pi}{2\pi}$$

$$\boxed{\sigma = 2}$$

Q.24 A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E} = E_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent n is _______.

ANS-2

Q.25 A solid material is found to have a temperature independent magnetic susceptibility, $\chi = C$. Which of the following statements is correct?

(A) If C is positive, the material is a diamagnet.
(B) If C is positive, the material is a ferromagnet.
(C) If C is negative, the material could be a type I superconductor.
(D) If C is positive, the material could be a type I superconductor.

ANS-(C)

Q.26 An infinite, conducting slab kept in a horizontal plane carries a uniform charge density σ. Another infinite slab of thickness t, made of a linear dielectric material of dielectric constant k, is kept above the conducting slab. The bound charge density on the upper surface of the dielectric slab is

(A) $\frac{\sigma}{2k}$ (B) $\frac{\sigma}{k}$

(C) $\frac{\sigma(k-2)}{2k}$ (D) $\frac{\sigma(k-1)}{k}$

$\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

$\vec{P} = \varepsilon\vec{E} - \varepsilon_0\vec{E}$

$= k\varepsilon_0\vec{E} - \varepsilon_0\vec{E}$

$= (k-1)\varepsilon_0\vec{E}$

$\vec{P} = \frac{(k-1)k\varepsilon_0\vec{E}}{k}$

$\vec{P} = \left(\frac{k-1}{k}\right)\sigma\hat{n}$ $[\sigma = k\varepsilon_0\vec{E}]$

$\sigma_b = \vec{P}\cdot\hat{n}$

$= \frac{(k-1)}{k}\sigma$

$\Rightarrow$ option (d)

Q.27 The number of spectroscopic terms resulting from the $L \cdot S$ coupling of a $3p$ electron and a $3d$ electron is _______.

ANS- use LS coupling configuration for Non-equivalent electron

Q.28 Which of the following statements is NOT correct?

(A) A deuteron can be disintegrated by irradiating it with gamma rays of energy 4 MeV.
(B) A deuteron has no excited states.
(C) A deuteron has no electric quadrupole moment.
(D) The $^1S_0$ state of deuteron cannot be formed.

ANS-(C)

Q.29 If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle$ for the ground state is

(A) $-\frac{3}{2}\hbar^2$ (B) $-\frac{3}{4}\hbar^2$ (C) 0 (D) $\frac{1}{4}\hbar^2$

:- $s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2}$

$$s = |\vec{s}_1 + \vec{s}_2| \text{ to } |\vec{s}_1 - \vec{s}_2|$$
$$= 1, 0$$

for ground state $s = 0$

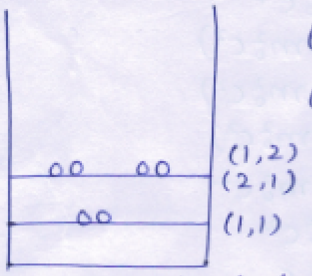
$$\vec{S} = \vec{S}_1 + \vec{S}_2$$
$$S^2 = (\vec{S}_1 + \vec{S}_2)\cdot(\vec{S}_1 + \vec{S}_2)$$
$$S^2 = S_1^2 + S_2^2 + 2\vec{S}_1\cdot\vec{S}_2$$
$$\vec{S}_1\cdot\vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$$
$$= \frac{1}{2}\left[s(s+1) - s_1(s_1+1) - s_2(s_2+1)\right]\hbar^2$$
$$= \frac{1}{2}\left[0 - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]\hbar^2$$
$$= -\frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2$$
$$\vec{S}_1\cdot\vec{S}_2 = -\frac{3}{4}\hbar^2$$

$\Longrightarrow$ option (b)

Q.30 A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0$ K. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is

:- No. of particles = 6

for electrons $(2s+1) = (2\times\frac{1}{2}+1) = 2$



$$E_{n_x n_y} = (n_x^2 + n_y^2)\frac{\pi^2\hbar^2}{2mL^2}$$
$$E_{(1,1)} = \frac{2\pi^2\hbar^2}{2mL^2}$$
$$E_{(1,2)} = \frac{5\pi^2\hbar^2}{2mL^2}$$

ground state energy

$$E = 2\times 2\left(\frac{\pi^2\hbar^2}{2mL^2}\right) + 4\times 5\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$
$$E = 24\left(\frac{\pi^2\hbar^2}{2mL^2}\right)$$

$\Longrightarrow$ Answer is 24.

Q.31 An alpha particle is accelerated in a cyclotron. It leaves the cyclotron with a kinetic energy of 16 MeV. The potential difference between the D electrodes is 50 kilovolts. The number of revolutions the alpha particle makes in its spiral path before it leaves the cyclotron is ______.

We know that for cyclotron

$K.E. = nqV_d$

then no. of revolution

$$n = \frac{K.E.}{qV_d}$$

$$= \frac{16\times10^6\,eV}{4e\times50\times10^3\,V}$$

$$\boxed{n = 80}$$

Q.32 Let V_i be the i^{th} component of a vector field $\vec{V}$, which has zero divergence. If $\partial_j \equiv \partial/\partial x_j$, the expression for $\epsilon_{ijk}\,\epsilon_{lmk}\,\partial_j\partial_l V_m$ is equal to

(A) $-\partial_j\partial_k V_i$ (B) $\partial_j\partial_k V_i$ (C) $\partial_j^2 V_i$ (D) $-\partial_j^2 V_i$

$$\varepsilon_{ijk}\varepsilon_{lmk}\partial_j\partial_l V_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\partial_j\partial_l V_m$$

$$= \delta_{il}\delta_{jm}\partial_j\partial_l V_m - \delta_{im}\delta_{jl}\partial_j\partial_l V_m$$

$$= 0 - \partial_j\cancel{\partial_i}\partial_j V_m$$

$$= -\partial_j^2 V_m$$

$\Longrightarrow$ option (d)

Q.33 The direction of $\vec{\nabla}f$ for a scalar field $f(x,y,z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$ at the point $P(1,1,2)$ is

(A) $\frac{\left(-\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (B) $\frac{\left(-\hat{j}+2\hat{k}\right)}{\sqrt{5}}$ (C) $\frac{\left(\hat{j}-2\hat{k}\right)}{\sqrt{5}}$ (D) $\frac{\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}$

$$f(x,y,z) = \frac{1}{2}x^2 - xy + \frac{1}{2}z^2$$

Point $(1,1,2)$

$$\vec{\nabla}f = \frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j} + \frac{df}{dz}\hat{k}$$

$$= (x-y)\hat{i} - x\hat{j} + z\hat{k}$$

$$\vec{\nabla}f\Big|_{(1,1,2)} = (1-1)\hat{i} - 1\hat{j} + 2\hat{k}$$

$$= -\hat{j} + 2\hat{k}$$

$$\text{Direction} = \frac{\vec{\nabla}f}{|\vec{\nabla}f|}$$

$$= \frac{1}{\sqrt{5}}(-\hat{j}+2\hat{k}) \Longrightarrow \text{option (b)}$$

Q.34 σ_x, σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y + \sigma_y\sigma_x$ is equal to

(A) $-3i\sigma_z$ (B) $-i\sigma_z$ (C) $i\sigma_z$ (D) $3i\sigma_z$

∵ we know that

$\sigma_x \sigma_y = i\sigma_z$

$\sigma_y \sigma_x = -i\sigma_z$

Now

$2\sigma_x \sigma_y + \sigma_y \sigma_x = 2i\sigma_z - i\sigma_z$

$= i\sigma_z$

$\Rightarrow$ option (C)

Q.35 A particle of mass $m = 0.1$ kg is initially at rest at origin. It starts moving with a uniform acceleration $\vec{a} = 10\hat{i}$ ms^{-2} at $t = 0$. The action S of the particle, in units of J-s, at $t = 2\ s$ is ______. (Give your answer upto two decimal places)

Sol:-

Action $S = \int_{t_1}^{t_2} L\,dt$

L is lagrangian

$L = T - V$

$\vec{a} = 10\hat{i}$

$v = u + at$

$v = 10t$ [∵ at initial $u = 0$]

$T = \frac{1}{2}mv^2 = \frac{1}{2}ma^2t^2$

$V = -F.x = -F\left[ut + \frac{1}{2}at^2\right]$

$= -F\left[\frac{1}{2}at^2\right]$

$= -ma\frac{1}{2}at^2 = -\frac{1}{2}ma^2t^2$

$L = ma^2t^2$

$S = \int_0^2 ma^2t^2\,dt = ma^2\left[\frac{t^3}{3}\right]_0^2$

$= 0.1 \times 100 \times \frac{8}{3}$

$\boxed{S = 26.66\ \text{J-s}}$

Q.36 A periodic function $f(x)$ of period 2π is defined in the interval $(-\pi < x < \pi)$ as:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$

The appropriate Fourier series expansion for $f(x)$ is

(A) $f(x) = (4/\pi)[\sin x + (\sin 3x)/3 + (\sin 5x)/5 + ...]$

(B) $f(x) = (4/\pi)[\sin x - (\sin 3x)/3 + (\sin 5x)/5 - ...]$

(C) $f(x) = (4/\pi)[\cos x + (\cos 3x)/3 + (\cos 5x)/5 + ...]$

(D) $f(x) = (4/\pi)[\cos x - (\cos 3x)/3 + (\cos 5x)/5 - ...]$

interval $(-\pi < x < \pi)$

$$f(x) = \begin{cases} -1 , & -\pi < x < 0 \\ 1 , & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -1\,dx + \frac{1}{\pi}\int_{0}^{\pi} 1\,dx$$

$$= \frac{-1}{\pi}\left[0+\pi\right] + \frac{1}{\pi}\left[\pi - 0\right] = 0$$

$a_n = 0$ [$\because$ odd function $f(-x) = -f(x)$]

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (-1)\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi} (1)\sin(nx)\,dx$$

$$= \frac{+1}{\pi}\left[\frac{\cos(nx)}{n}\right]_{-\pi}^{0} + \frac{-1}{\pi}\left[\frac{\cos(nx)}{n}\right]_{0}^{\pi}$$

$$= \frac{1}{n\pi}\left[1-(-1)^n\right] - \frac{1}{n\pi}\left[(-1)^n - 1\right]$$

$$= \frac{2}{n\pi}\left(1-(-1)^n\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$= 0 + 0 + \frac{2}{\pi}\sum_{n=1}\left(1-(-1)^n\right)\frac{\sin(nx)}{n}$$

$$= \frac{2}{\pi}\left[\frac{2}{1}\sin(x) + 0 + \frac{2}{3}\sin(3x) + \cdots\right]$$

$$f(x) = \frac{4}{\pi}\left(\sin x + \sin 3x + \cdots\right)$$

$\Rightarrow$ option ⓐ

Q.37 Atoms, which can be assumed to be hard spheres of radius R, are arranged in an *fcc* lattice with lattice constant a, such that each atom touches its nearest neighbours. Take the center of one of the atoms as the origin. Another atom of radius r (assumed to be hard sphere) is to be accommodated at a position $(0, a/2, 0)$ without distorting the lattice. The maximum value of r/R is ________. (Give your answer upto two decimal places)

ANS-0.41

Q.38 In an inertial frame of reference S, an observer finds two events occurring at the same time at co-ordinates $x_1 = 0$ and $x_2 = d$. A different inertial frame S' moves with velocity v with respect to S along the positive x-axis. An observer in S' also notices these two events and finds them to occur at times t_1' and t_2' and at positions x_1' and x_2', respectively. If $\Delta t' = t_2' - t_1'$, $\Delta x' = x_2' - x_1'$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which of the following statements is true ?

(A) $\Delta t' = 0, \Delta x' = \gamma d$

(B) $\Delta t' = 0, \Delta x' = d/\gamma$

(C) $\Delta t' = -\gamma \mathrm{v} d/c^2, \Delta x' = \gamma d$

(D) $\Delta t' = -\gamma \mathrm{v} d/c^2, \Delta x' = d/\gamma$

:- We know that

$t' = \left(t - \frac{vx}{c^2}\right)\gamma$

$x' = \gamma(x - vt)$

Now $t_1' = (t-0)\gamma \quad [\because x_1 = 0]$

$t_1' = \gamma t$

$t_2' = \left(t - \frac{vd}{c^2}\right)\gamma \quad [\because x_2 = d]$

$x_1' = \gamma(0 - vt)$

$x_1' = -\gamma vt$

$x_2' = \gamma(d - vt)$

$\Delta x' = x_2' - x_1'$

$= \gamma d$

$\Delta t' = t_2' - t_1'$

$= \left(\gamma t - \frac{\gamma vd}{c^2}\right) - \gamma t$

$\Delta t' = -\gamma\frac{vd}{c^2}$

$\Rightarrow$ option (c)

Q.39 The energy vs. wave vector $(E-k)$ relationship near the bottom of a band for a solid can be approximated as $E = A(ka)^2 + B(ka)^4$, where the lattice constant a = 2.1 Å. The values of A and B are 6.3×10^{-19} J and 3.2×10^{-20} J, respectively. At the bottom of the conduction band, the ratio of the effective mass of the electron to the mass of free electron is _______. (Give your answer upto two decimal places)
(Take $\hbar$ =$1.05\text{x}10^{-34}$ J-s, mass of free electron= $9.1\text{x}10^{-31}$ kg)

:- $E(k) = A(ka)^2 + B(ka)^4$

at the bottom

$E(k) = Aa^2k^2$

$\frac{d^2E}{dk^2} = 2a^2A$

$m^* = \frac{\hbar^2}{d^2E/dk^2} = \frac{\hbar^2}{2a^2A}$

$= \frac{(1.05\times10^{-34})^2}{2\times(2.1)^2\times10^{-20}\times6.3\times10^{-19}}$

$= 0.0199\times10^{-29}$

$\frac{m^*}{m} = \frac{0.0199\times10^{-29}}{9.1\times10^{-31}} = 0.22$

Q.40 The electric field component of a plane electromagnetic wave travelling in vacuum is given by $\vec{E}(z,t) = E_0 \cos(kz - \omega t)\hat{i}$. The Poynting vector for the wave is

(A) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{j}$

(B) $(c\varepsilon_0/2)E_0^2 \cos^2(kz-\omega t)\hat{k}$

(C) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{j}$

(D) $c\varepsilon_0 E_0^2 \cos^2(kz-\omega t)\hat{k}$

$\vec{E}(z,t) = E_0 \cos(kz-\omega t)\hat{i}$

$\vec{k} = k\hat{z} = k\hat{k}$

$\vec{B} = \frac{1}{c}(\hat{k}\times\vec{E})$

$= \frac{E_0}{c}\cos(kz-\omega t)(\hat{k}\times\hat{i})$

$= \frac{E_0}{c}\cos(kz-\omega t)\hat{j}$

$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$

$= \frac{E_0}{c\,\mu_0}E_0\cos^2(kz-\omega t)(\hat{i}\times\hat{j})$

$= \frac{1}{c\mu_0}E_0^2\cos^2(kz-\omega t)\hat{k}$

$= \frac{\varepsilon_0}{\varepsilon_0\mu_0 c}E_0^2\cos^2(kz-\omega t)\hat{k}$

$= \frac{c^2\varepsilon_0}{c}E_0^2\cos^2(kz-\omega t)\hat{k}$

$\vec{S} = c\varepsilon_0 E_0^2\cos^2(kz-\omega t)\hat{k}$

$\Longrightarrow$ option (d)

Q.41 Consider a system having three energy levels with energies 0, 2ε and 3ε, with respective degeneracies of 2, 2 and 3. Four bosons of spin zero have to be accommodated in these levels such that the total energy of the system is 10ε. The number of ways in which it can be done is ________.

Energy levels → $0, 2\varepsilon, 3\varepsilon$

degeneracies → 2, 2, 3

3ε — (2) — 3

2ε — (2) — 2

0 — — 2

$\Omega = \prod_i \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!}$

$= \frac{(2+2-1)!}{2!(2-1)!} \times \frac{(2+3-1)!}{2!(3-1)!}$

$= \frac{3!}{2} \times \frac{4!}{4}$

$= 3\times 6$

$\boxed{\Omega = 18}$

Q.42 The Lagrangian of a system is given by

$$L = \frac{1}{2} m l^2 \left[\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right] - mgl\cos\theta,$$ where m, l and g are constants.

Which of the following is conserved?

(A) $\dot{\varphi}\sin^2\theta$ (B) $\dot{\varphi}\sin\theta$ (C) $\frac{\dot{\varphi}}{\sin\theta}$ (D) $\frac{\dot{\varphi}}{\sin^2\theta}$

$L = \frac{1}{2} m l^2 [\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2] - mgl\cos\theta$

Here ϕ is cyclic cordinate

$\frac{\partial L}{\partial \dot{\phi}}$ = conserved

$\frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2} m l^2 . 2\dot{\phi}\sin^2\theta$

$= m l^2 \dot{\phi}\sin^2\theta$

conserved $\rightarrow \dot{\phi}\sin^2\theta$

$\Longrightarrow$ option (a)

Q.43 Protons and α-particles of equal initial momenta are scattered off a gold foil in a Rutherford scattering experiment. The scattering cross sections for proton on gold and α-particle on gold are σ_p and σ_α respectively. The ratio σ_α/σ_p is ________.

we know that

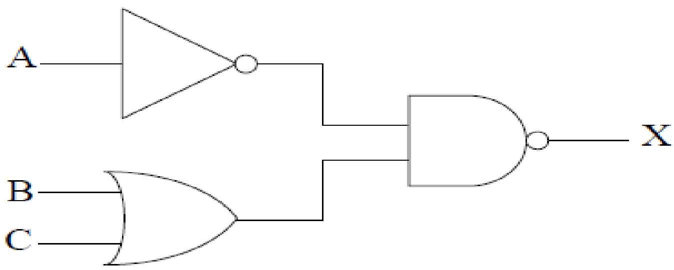
$\sigma \propto m^2$

$m_\alpha = 2m_p$

$\frac{\sigma_\alpha}{\sigma_p} = \left(\frac{m_\alpha}{m_p}\right)^2 = \left(\frac{2m_p}{m_p}\right)^2$

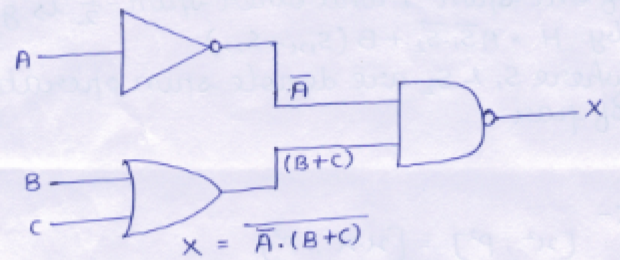
$\boxed{\frac{\sigma_\alpha}{\sigma_p} = 4}$

Q.44 For the digital circuit given below, the output X is



(A) $\overline{\overline{A} + B\cdot C}$ (B) $\overline{\overline{A}\cdot(B+C)}$

(C) $\overline{A}\cdot(B+C)$ (D) $A + \overline{(B\cdot C)}$



$X = \overline{\bar{A}\cdot(B+C)}$

$\Rightarrow$ option (b)

Q.45 The Fermi energies of two metals X and Y are 5 eV and 7 eV and their Debye temperatures are 170 K and 340 K, respectively. The molar specific heats of these metals at constant volume at low temperatures can be written as $(C_V)_X = \gamma_X T + A_X T^3$ and $(C_V)_Y = \gamma_Y T + A_Y T^3$, where γ and A are constants. Assuming that the thermal effective mass of the electrons in the two metals are same, which of the following is correct?

(A) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = 8$

(B) $\frac{\gamma_X}{\gamma_Y} = \frac{7}{5}, \frac{A_X}{A_Y} = \frac{1}{8}$

(C) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = \frac{1}{8}$

(D) $\frac{\gamma_X}{\gamma_Y} = \frac{5}{7}, \frac{A_X}{A_Y} = 8$

We know that

$\gamma \propto \frac{1}{E_f}$ (for e^-)

$A \propto \frac{1}{(T_D)^3}$ (for phonon)

$\gamma_x \propto \frac{1}{5}$, $\gamma_y \propto \frac{1}{7}$

$A_x \propto \frac{1}{(170)^3}$, $A_y \propto \frac{1}{(340)^3}$

$\frac{\gamma_x}{\gamma_y} = \frac{7}{5}$

$\frac{A_x}{A_y} = \left(\frac{340}{170}\right)^3 = (2)^3 = 8$

$\Rightarrow$ option (a)

Q.46 A two-level system has energies zero and E. The level with zero energy is non-degenerate, while the level with energy E is triply degenerate. The mean energy of a classical particle in this system at a temperature T is

(A) $\frac{Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

(B) $\frac{Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$

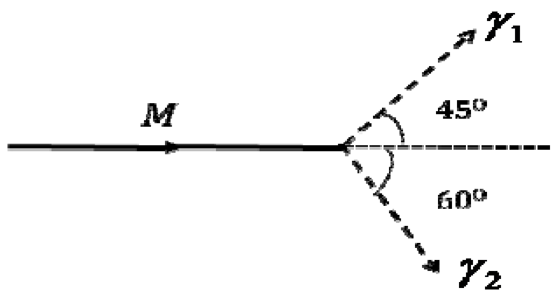
(C) $\frac{3Ee^{-E/k_BT}}{1+e^{-E/k_BT}}$

(D) $\frac{3Ee^{-E/k_BT}}{1+3e^{-E/k_BT}}$

$$Z = \sum_i g_i e^{-\beta \varepsilon_i}$$
$$Z = 1 \times e^{-\beta \times 0} + 3 e^{-\beta \varepsilon}$$
$$Z = 1 + 3 e^{-\beta \varepsilon}$$
$$\langle \varepsilon \rangle = -\frac{\partial}{\partial \beta}(\ln Z)$$
$$= -\frac{\partial}{\partial \beta}\left[\ln(1+3e^{-\beta \varepsilon})\right]$$
$$= -\left(\frac{-3\varepsilon e^{-\beta \varepsilon}}{(1+3e^{-\beta \varepsilon})}\right)$$
$$\langle \varepsilon \rangle = \frac{3\varepsilon e^{-\beta \varepsilon}}{1+3e^{-\beta \varepsilon}}$$

$\Rightarrow$ option (d)

Q.47 A particle of rest mass M is moving along the positive x-direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is 1 GeV and the energy of γ_2 is 0.82 GeV. The value of M (in units of GeV/c^2) is ________. (Give your answer upto two decimal places)



momentum conservation

$$P_M = P_{\gamma_1} + P_{\gamma_2}$$
$$= \frac{E_{\gamma_1}}{c}\cos 45^\circ + \frac{E_{\gamma_2}}{c}\cos 60^\circ$$
$$= \frac{1}{c}\frac{1}{\sqrt{2}} + \frac{0.82}{c}\frac{1}{2}$$
$$P_M = \frac{0.707}{c} + \frac{0.41}{c}$$
$$P_M = 1.117\ \text{GeV}/c$$

Now

$$M^2 = \frac{E_T^2 - P_M^2 c^2}{c^4}$$
$$= \frac{(1.82)^2 - (1.117)^2}{c^4}$$
$$= \frac{3.3124 - 1.2476}{c^4}$$
$$= \frac{2.0648}{c^4}$$
$$M = 1.43\ \text{GeV}/c^2$$

Q.48 If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $[x^2, p^2]$ is

(A) $i\hbar(xp - px)$ (B) $2i\hbar(xp - px)$
(C) $i\hbar(xp + px)$ (D) $2i\hbar(xp + px)$

$[x^2, p^2] = [x \cdot x, p^2]$

$= x[x, p^2] + [x, p^2]x$

$= x(2i\hbar p) + (2i\hbar p)x$

$\left[\because [x, f(p)] = \frac{\partial f(p)}{\partial p}[x, p]\right]$

$= 2i\hbar(xp + px)$

$\Rightarrow$ option (d)

Q.49 The x-y plane is the boundary between free space and a magnetic material with relative permeability μ_r. The magnetic field in the free space is $B_x\hat{i} + B_z\hat{k}$. The magnetic field in the magnetic material is

(A) $B_x\hat{i} + B_z\hat{k}$ (B) $B_x\hat{i} + \mu_r B_z\hat{k}$
(C) $\frac{1}{\mu_r}B_x\hat{i} + B_z\hat{k}$ (D) $\mu_r B_x\hat{i} + B_z\hat{k}$

$\vec{B}_1 = B_x\hat{i} + B_z\hat{k}$

Boundary is x-y plane

$\vec{B}_{1n} = B_z\hat{k}$

$\vec{B}_{1t} = B_x\hat{i}$

from boundary condition

$\vec{B}_{1n} = \vec{B}_{2n}$

$\Rightarrow \vec{B}_{2n} = B_z\hat{k}$

And $\vec{H}_{1t} = \vec{H}_{2t}$

$\frac{\vec{B}_{1t}}{\mu_0} = \frac{\vec{B}_{2t}}{\mu_0\mu_r}$

$\vec{B}_{2t} = \mu_r\vec{B}_{1t}$

$\vec{B}_{2t} = \mu_r B_x\hat{i}$

$\Rightarrow \vec{B}_2 = \mu_r B_x\hat{i} + B_z\hat{k}$

$\Rightarrow$ option (d)

Q.50 Let $|l,m\rangle$ be the simultaneous eigenstates of L^2 and L_z. Here $\vec{L}$ is the angular momentum operator with Cartesian components (L_x, L_y, L_z), l is the angular momentum quantum number and m is the azimuthal quantum number. The value of $\langle 1,0|(L_x + iL_y)|1,-1\rangle$ is

(A) 0 (B) $\hbar$ (C) $\sqrt{2}\hbar$ (D) $\sqrt{3}\hbar$

Sol :-

$L_+ = L_x + iL_y$

$L_+|l,m\rangle = \sqrt{l(l+1)-m(m+1)}\hbar|l,m+1\rangle$

Now

$\langle 1,0|L_x+iL_y|1,-1\rangle = \langle 1,0|L_+|1,-1\rangle$

$= \langle 1,0|\sqrt{1(1+1)-(-1)(-1+1)}\hbar|1,0\rangle$

$= \langle 1,0|\sqrt{2}\hbar|1,0\rangle$

$= \sqrt{2}\hbar \langle 1,0|1,0\rangle$

$= \sqrt{2}\hbar$

$\Longrightarrow$ option (c)

Q.51 For the parity operator P, which of the following statements is NOT true?

(A) $P^\dagger = P$ (B) $P^2 = -P$ (C) $P^2 = I$ (D) $P^\dagger = P^{-1}$

We know that, parity operator

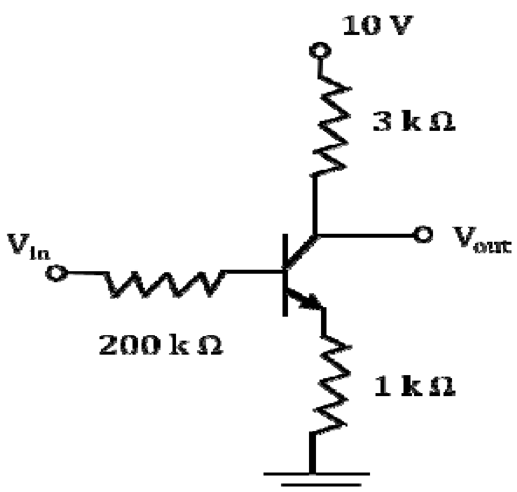
$P^2 = P$, $P = P^{-1}$

$P^\dagger = P = P^{-1}$

$P^2 = I$

$\Longrightarrow$ option (b)

Q.52 For the transistor shown in the figure, assume $V_{BE} = 0.7$ V and $\beta_{dc} = 100$. If $V_{in} = 5$V, V_{out} (in Volts) is __________. (Give your answer upto one decimal place)



sol^n:-

$V_{in} = 200I_B + V_{BE} + I_E$

$5 = 200I_B + 0.7 + I_B + I_C$

$4.3 = 200I_B + I_B + 100I_B$

$4.3 = 301\,I_B$

$I_B = 0.0143\,mA$

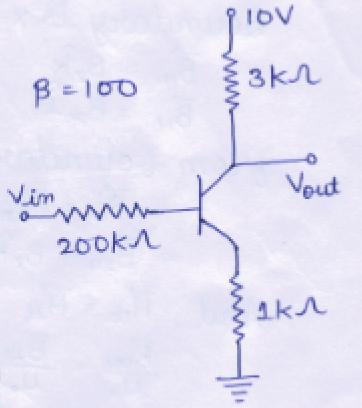
Now $I_C = \frac{10 - V_{out}}{3}$

$3\beta I_B = 10 - V_{out}$

$V_{out} = 10 - 300I_B$

$= 10 - 4.29$

$V_{out} = 5.71$ volts



Q.53 The state of a system is given by

$$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$$

where $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is ________. (Give your answer upto two decimal places)

sol :-

$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$

Normalisation

$|A|^2 + 4|A|^2 + 9|A|^2 = 1$

$14|A|^2 = 1$

$|A| = \frac{1}{\sqrt{14}}$

Now $|\psi\rangle = \frac{1}{\sqrt{14}}|\phi_1\rangle + \frac{2}{\sqrt{14}}|\phi_2\rangle + \frac{3}{\sqrt{14}}|\phi_3\rangle$

Probability of finding system in state $|\phi_2\rangle = |\langle\phi_2|\psi\rangle|^2$

$= \left|\frac{2}{\sqrt{14}}\right|^2$

$P = \frac{4}{14} = 0.28$

Q.54 According to the nuclear shell model, the respective ground state spin-parity values of $^{15}_{8}O$ and $^{17}_{8}O$ nuclei are

(A) $\frac{1}{2}^+, \frac{1}{2}^-$

(B) $\frac{1}{2}^-, \frac{5}{2}^+$

(C) $\frac{3}{2}^-, \frac{5}{2}^+$

(D) $\frac{3}{2}^-, \frac{1}{2}^-$

:- ${}^{15}_{8}O \Rightarrow p = 8 \ , n = 7$

$n = 7 \Rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1$

$\Downarrow$

$J = 1/2 \ , l = 1$

$P = (-1)^l = -1$

$\Rightarrow J^P = \frac{1}{2}^-$

${}^{17}_{8}O \Rightarrow p = 8 \ , n = 9$

$n = 9 \Rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

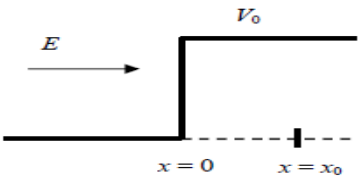
$\Downarrow$

$J = \frac{5}{2} \ , l = 2$

$P = (-1)^l = +1$

$\Rightarrow J^P = \frac{5}{2}^+ \Rightarrow$ option (b)

Q.55 A particle of mass m and energy E, moving in the positive x direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0, where $V_0 > E$. At $x = x_0$, where $x_0 > 0$, the probability of finding the electron is $1/e$ times the probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is



(A) $\frac{2}{\alpha}$ (B) $\frac{1}{\alpha}$ (C) $\frac{1}{2\alpha}$ (D) $\frac{1}{4\alpha}$

We know that

$\psi_{II} = e^{-\alpha x} \quad [\because V_0 > E]$

$|\psi|^2 = \psi^* \psi = e^{-2\alpha x}$

Now at $x = x_0$

$|\psi|^2 = \frac{1}{e} = e^{-2\alpha x_0}$

$\Rightarrow e^{-1} = e^{-2\alpha x_0}$

$\Rightarrow 2\alpha x_0 = 1$

$x_0 = \frac{1}{2\alpha}$

$\Rightarrow$ option (c)

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