CSIR-NET Full length TEST PAPER

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ATS-2

(Full Length-Q.M. & ED.)

PART-A

1. If each side of a square is increased by 25%, find the percentage change in its area?
   (a)65.25 (b)56.25
   (c)65 (d)56
2. A bag contains 6 white and 4 black balls .2 balls are drawn at random. Find the probability that they are of same colour.
   (a)1/2 (b)7/15
   (c)8/15 (d)1/9
3. If 20% of a = b, then b% of 20 is the same as :
   (a) 4% of a (b) 6% of a
   (c) 8% of a (d) 10% of a
4. A problem is given to three students whose chances of solving it are 1/2, 1/3 and 1/4 respectively. What is the probability that the problem will be solved?
   (a)1/4 (b)1/2
   (c)3/4 (d)7/12
5. If each edge of a cube is increased by 50%, find the percentage increase in Its surface area
   (a) 125% (b) 150%
   (c)175% (d)110%
6. A started a business by investing Rs. 48,500. After 3 months B joined him with a capital of Rs. 50,000. After 6 months C joined them with a capital of Rs.52,500. At the end of the year they earned a profit of Rs. 10,776. What will be B's share in profit?
   (a)350 (b)3750
   (c)3600 (d)3700
7. Direction : Read the following graph carefully and answer the question given below.Data related production of grams in five different states in six different years (in tonnes)

	2008	2009	2010	2011	2012	2013
P	4.4	3.2	3.8	4.6	5	2.8
Q	2.4	4.6	2.4	3	4.2	3.4
R	5	4.4	3.2	5.6	2.6	4.2
S	1.4	4	2.4	1.8	3.6	5.4
T	3.8	2.2	4.6	2.4	1.6	2

What is the percentage increase in production of grams in state Q in 2012 as compared to the previous year?

(a)40% (b)38%
(c)35% (d)37%

8 An accurate clock shows 8 o'clock in the morning. Through how may degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?

(a)360 (b)180
(c)90 (d)60

9. SIGNAL is coded as TKJOCO, then what will be the code for CALENDER ?

(a) DCOEQFHF (b) EDOFPEHU
(c) ECOEQFGT (d) DCOFPGFT

10 N is a four digit number. If the leftmost digit is removed, the resulting three digit number is 1/9th Of N. How many such N are possible ?

(a)10 (b)9
(c)8 (d)7

11. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?

(a) 54 past 4 (b) (53 + 7/11) past 4
(c) (54 + 8/11) past 4 (d) (54 + 6/11) past 4

12. If South-East becomes North, North-East becomes West and so on. What will West become?

(a) North-East (b) North-West

(c) South-East (d) South-West

13. In a certain code language COMPUTER is written as RFUVQNPC. How will MEDICINE be written in that code language?

(a) MFEDJJOE (b) EOJDEJFM

(c) MFEJDJOE (d) EOJDJEFM

14. By selling 45 lemons for Rs 40, a man loses 20%. How many should he sell for Rs 24 to gain 20% in the transaction?

(a) 16 (b) 18

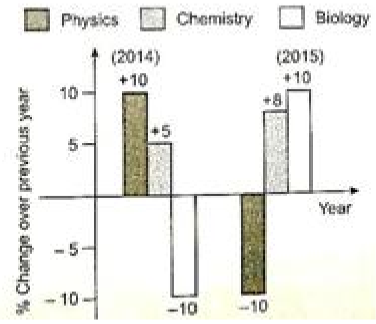
(c) 20 (d)22

15. Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

(a) 40 (b) 80

(c) 120 (d)200

16. Which of the following inferences can be drawn from the above graph ?



(a) The total number of students qualifying in Physics in 2015 and 2014 is the same

(b) The number of students qualifying in Biology in 2015 is less than that in 2013

(c) The number of Chemistry students qualifying in 2015 must be more than the number of students who qualified in Biology in 2014

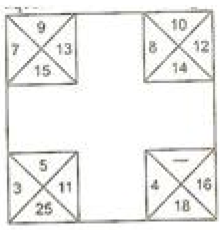
(d) The number of students qualifying in Physics in 2015 is equal to the number of students in Biology that qualified in 2014

17. AB and CD are two chords of a circle subtending 60° and 120° respectively at the same point on the circumference of the circle. Then AB : CD is—

(a) 3:1 (b) 2:1

(c) 1:1 (d) 3X2

18. The relationship among the numbers in each corner square is the same as that in the other corner squares. Find the missing number—



(a) 10 (b) 8

(c) 6 (d)12

19. A student appearing for an exam is declared to have failed the exam if his/her score is less than half the median score. This implies—

(a) 1/4 of the students appearing for the exam always fail
(b) if a student scores less than 1/4 of the maximum score, he/she always fails
(c) if a student scores more than 1/2 of the maximum score, he/she always passes
(d) it is possible that no one fails

20. When an ideal monoatomic gas is expanded adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio T/T_0 is—

(a) 13 (b) 132/3
(c) 131/3 (d) 3

PART-B

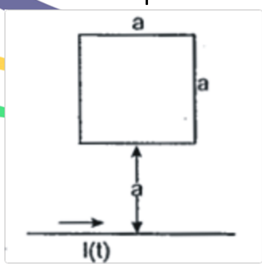
21. Consider a neutral conducting sphere. A positive point charge q is placed outside the sphere. The net charge on the sphere is then
(a) -q and distributed uniformly over the surface of the sphere
(b) -q and appears only at the point on the sphere closest to the point charge.
(c) - q and distributed non-uniformly over the entire surface.
(d) Zero

22. Two Points charges + q and – q are kept at (0, 0, a) and (0, 0, -a) respectively. Work Done by the electric field when another charge +q is moved from (-a, 0, 0) to (a, 0, 0) is
(a) Positive (b) Negative
(c) Zero
(d) depends on the path joining initial and final position.

23. The electrostatic potential inside a charged sphere is given by $\phi = ar^2 + b$ where r is the distance form the centre a and b are constants. The charge density inside the sphere is
(a) $-6a\,\varepsilon_0$ (b) $-24\pi a\,\varepsilon_0 r$
(c) $-6a\,\varepsilon_0 r$ (d) $-24\pi a\,\varepsilon_0$

24. A 1 w/m² uniform plane in vacuum $\hat{x} E_t \cos(wt - kz)$ is normally incident upon a planar dielectric with $\varepsilon = 4\varepsilon_0$. the intensity of reflected wave is
(a) 111 mw/m² (b) 222 mw/m²
(c) 333 mw/m² (d) 444 mw/m²

25. An infinitely long wire carrying a current $I(t) = I_0 \cos(\omega t)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is:



(a) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R} ln 2$ (b) $\frac{\mu_0}{\pi}\frac{aI_0\omega}{R} n 2$

(c) $\frac{2\mu_0}{\pi}\frac{aI_0\omega}{R} ln 2$ (d) $\frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}$

25. A current carrying loop lying in the plane of the paper is in the shape of an equilateral triangle of side a. It carries a current I in the clockwise sense. If $\hat{k}$ denotes the outward normal to the plane of the paper, the magnetic moment m due to the current loop is:

(a) $\vec{m} = a^2 I\hat{k}$

(b) $\vec{m} = -\frac{1}{2}a^2 I\hat{k}$

(c) $\vec{m} = \frac{\sqrt{3}}{2}a^2 I\hat{k}$

(d) $\vec{m} = -\frac{\sqrt{3}}{2}a^2 I\hat{k}$

26. Two semi-infinite grounded conducting planes meet at an angle 60^o the number of image charges formed:

(a) 4
(b) 3
(c) 5
(d) 6

27. The skin depth (δ) in aluminium (Al) with conductivity $\sigma=(3.6\times10^7)\Omega^{-1}\,m^{-1}$ at 1.6MHz is.

(a) 66.4 μm
(b) 54.5 μm
(c) 74.4 μm
(d) 46.9 μm

28. A Parallel plate capacitor with square plates of side 1 m separated by 1 micro meter is filled with a medium of dielectric constant of 10. If the charges on the two plates are 1 C and -1 C, the voltage across the capacitor is .... kV. (up to two decimal places). ($\varepsilon_0=8.854\times10^{-12}$F/m) .

29. An infinite solenoid carries a time varying current $I(t)=At^2$, with $A\neq0$. The axis of the solenoid is along the $\hat{z}$ direction. $\hat{r}$ and $\hat{\theta}$ are the usual radial and polar directions in cylindrical polar coordinates. $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z}$ is the magnetic field at a point outside the solenoid. Which one of the following statements is true?

(a) $B_r = 0, B_\theta=0, B_z=0$
(b) $B_r \neq0, B_\theta\neq0, B_z=0$
(c) $B_r\neq0, B_\theta \neq0, B_z\neq0$
(d) $B_r=0, B_\theta=0, B_z\neq0$

30. Lagrangian of a system is $L = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) - \frac{1}{2}k(x^2+y^2+z^2)$. If a uniform electric field is applied in z-direction then which of the following remains conserved.

(a) L_x
(b) L_y
(c) L_z
(d) All L_x, L_y, L_z

Here L_x, L_y, L_z are Cartesian components of angular momentum.

31. A Source of light is moving with speed $\frac{c}{2}$ towards an observer who is also moving towards the source with speed $\frac{c}{2}$. Both the velocities are with respect to lab. If the source emits light of frequency ν_0 then frequency measured by the observer will be

(a) ν_0
(b) $\frac{\sqrt{3}\nu_0}{2}$
(c) $\sqrt{3}\nu_0$
(d) $3\nu_0$

32. A binary system of star consists of two stars of masses m and 3m revolving about their common centre of mass under their own mutual gravitational attraction. Ration of angular momentum of first to the second is.

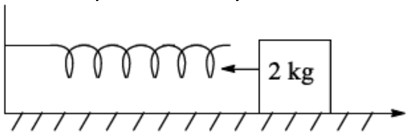
(a) $\frac{1}{3}$
(b) 3
(c) 1
(d) 9

33. Lagrangian of a system is $L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)-K(2x+3y-z)$. Which of the following is not constant of motion.

(a) $3p_x-2p_y$ (b) $3p_x+p_z$

(c) $2p_x+p_z$ (d) p_y+3p_z

34. A 2kg block travelling with speed of 5 m/sec collides with a spring of spring constant 2 N/m and compressed it by 3 m.



What is the coefficient of kinetic friction between the block and surface.

(a) 0.57 (b) 0.27

(c) 0.12 (d) 0.012

35. A spaceship is travelling with a velocity of 0.7c away from a space station. The spaceship ejects a probe with a velocity 0.59c opposite to its own velocity. A person in the space station would see the probe moving at a speed Xc, where the value of X is .... (up to three decimal places).

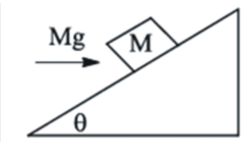
36. In the special theory of relativity, consider a Lorentz boost by a velocity 'v' along the x-direction. If u=ct+x, then the boosted value u'=ct'+x' is:

(a) $\mu'=\sqrt{\frac{1+v/c}{1-v/c}}u$ (b) $u'=\sqrt{\frac{1-v/c}{1+v/c}}u$

(c) $u'=\frac{1+v/c}{1-v/c}u$ (d) $u'=\frac{1-v/c}{1+v/c}u$

37. A block of mass M rests on a plane inclined art angle θ below figure. You apply a horizontal force of Mg to the block, as shown.

Let the coefficient of static friction be μ. For what range of angles will the block remain still?



(a) $\frac{1-\mu}{1+\mu}\le\tan\theta\le\frac{1+\mu}{1-\mu}$ (b) $\frac{1+\mu}{1-\mu}\le\tan\theta\le\frac{1-\mu}{1+\mu}$

(c) $\frac{1-\mu}{1+\mu}\ge\tan\theta\ge\frac{1+\mu}{1-\mu}$ (d) $\frac{1+\mu}{1-\mu}\ge\tan\theta\ge\frac{1-\mu}{1+\mu}$

38. The Lagrangian of a simple pendulum of length 1 and mass m is given by

$L(\theta,\dot{\theta})=\frac{1}{2}m\left[l^2\dot{\theta}^2-gl\theta^2\right]$ which of the following statements are correct?

(1) Its Langrange's caution of motion is $\ddot{\theta}+\frac{g}{l}\theta=0$

(2) its generalized momentum is $P_\theta = ml^2\dot{\theta}$

(3) Its Hamiltonian is $H = \frac{1}{2}\left[\frac{P_\theta^2}{ml^2} + mgl\theta^2\right]$

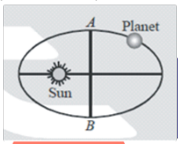
(4) It has two degrees of freedom.

(a) 1, 2 and 4
(b) 1, 3 and 4
(c) 1, 2 and 3
(d) 1, 2, 3 and 4

39. Consider a particle of mass m moving with speed V. If T_R denotes the relativistic kinetic energy and T_N its non-relativistic approximation, then value of $\frac{T_R - T_n}{T_R}$ for V=0.001 C is.

(a) 1.25×10^{-5}
(b) 5×10^{-5}
(c)) 7.5×10^{-5}
(d) 10×10^{-4}

40. A planet revolves around the sun in an elliptical orbit of eccentricity 'e'. If T_1 and T_2 be time taken by planet in going from one extreme of minor axis to the other extreme (i.e. from A to B) through perigee and apogee respectively then value of T_1/T_2 is



(a) $\frac{1-e}{1+e}$
(b) $\frac{1+e}{1-e}$
(c) $\frac{\pi-e}{\pi+e}$
(d) 1

41. A planet p_1 revolves around a star S_1 (mass M) in an elliptical orbit of semi-major axis 'a' with time of revolution T_1. Another Planet p_2 revolves around another star S_2 (mass 32M) in an elliptical orbit of semi-major axis '2a' time of revolution T_2. Value of T_2/T_1 is

(a) $2\sqrt{2}$
(b) 2
(c) $\frac{1}{2}$
(d) 1

42. A glass slab having refractive index 2 is moving with velocity $\frac{c}{2}\hat{i}$ with respect to lab. A light beam is travelling inside the slab in y-direction. What is speed of light beam as measured in lab frame?

(a) $\frac{\sqrt{7}}{4}c$
(b) $\frac{c}{\sqrt{2}}$
(c) $\frac{\sqrt{2}c}{3}$
(d) $\frac{c}{2\sqrt{2}}$

43. A particle of rest mass m_0 moves with constant speed c/2 at angle 60° to +x axis with respect to lab frame.

What is energy of particle in a frame which is moving with velocity c/2 in +x direction

(a) $2m_0c^2$ (b) $\frac{\sqrt{3}m_0c^2}{2}$

(c) $\frac{3m_0c^2}{2}$ (d) $\frac{7m_0c^2}{6}$

44. A particle is moving in one dimensional potential $V(x)=kx^6$. If amplitude is doubled then time period of its motion will become.

(a) $\frac{1}{2}$ times (b) 2 times

(c) $\frac{1}{4}$ times (d) 4 times

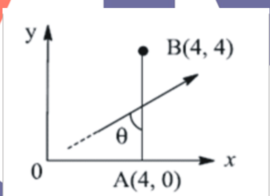
45. A thin rod of mass m and length *l* has linear mass density proportional to distance from one end. Moment of inertia of the rod about an axis passing through center and perpendicular to length is.

(a) $\frac{ml^2}{12}$ (b) $\frac{ml^2}{4}$

(c) $\frac{ml^2}{3}$ (d) $\frac{ml^2}{24}$

PART-C

46. The surface current density in the xoy plane is given by $k(x,y)=k_0\left(\frac{y^2}{b^2}\hat{i}+\frac{x^2}{a^2}\hat{j}\right)$ calculate the rate of flow of charge across the straight line AB where A is (4.0) and B is the point (4,4) as shown in fig.



(a) $\frac{64k_0}{3}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$ (b) $\frac{64k_0}{3}\left(\frac{1}{a^2}-\frac{1}{b^2}\right)$

(c) $\frac{64}{3}\frac{k_0}{b^2}$ (d) $\frac{64}{3}\frac{k_0}{a^2}$

47. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ corresponding to the scalar and vector potentials, $V(x,y,z,t)=0$ and $\vec{A}(x,y,z,t)=\frac{1}{2}\hat{k}\mu_0A_0(ct-x)$, Where A_0 is a constant, are

(a) $\vec{E}=0$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0A_0$ (b) $\vec{E}=-\frac{1}{2}\hat{k}\mu_0A_0c$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0A_0$

(c) $\vec{E}=0$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0A_0$ (d) $\vec{E}=\frac{1}{2}\hat{k}\mu_0A_0c$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0A_0$

48. In the region far from a source, the time dependent electric field at a point (r, θ, φ) is

$$\vec{E}(r,\theta,\phi)=\hat{\phi}E_0\omega^2\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]$$

Where ω is angular frequency of the source. The total power radiated (averaged over a cycle) is

(a) $\frac{2\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (b) $\frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$

(c) $\frac{4}{3\pi}\frac{E_0^2\omega^4}{\mu_0 c}$ (d) $\frac{2}{3}\frac{E_0^2\omega^4}{\mu_0 c}$

49. Which of the following represents an elliptically polarized light?

(a) $E_0(\hat{x}+i\hat{y})e^{i(\bar{k}\cdot\bar{r}-\omega t)}$ (b) $E_0(\hat{x}-i\hat{y})e^{i(\bar{k}\cdot\bar{r}-\omega t)}$

(c) $(E_1\hat{x}+iE_2\hat{y})e^{i(\bar{k}\cdot\bar{r}-\omega t)}$ (d) $(E_1\hat{x}+E_2\hat{y})e^{i(\bar{k}\cdot\bar{r}-\omega t)}$

50. Eight identical charge each of value +q, are fixed at the eigth corners of a cube side a. Calculated the electrostatic potential at the centre of a side.

(a) $\frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{3}+\frac{1}{\sqrt{5}}\right)$ (b) $\frac{q}{4\pi\varepsilon_0 a}\left(\sqrt{2}+\frac{\sqrt{2}}{\sqrt{3}}\right)$

(c) $\frac{4q}{\pi\varepsilon_0 a\sqrt{3}}$ (d) $\frac{2q}{\pi\varepsilon_0 a}\left(\frac{2}{3}+\frac{1}{\sqrt{5}}\right)$

51. The magnitude of electric field inside a sphere of radius R is given by

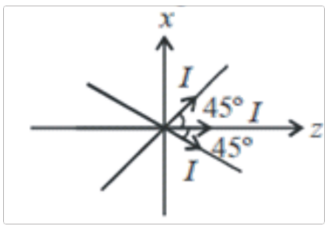
$$E=\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}+\frac{r^2}{4R}\right)$$

The electrostatics potential at a distance 2R from the centre of the sphere is given by

(a) $\frac{2\rho_0 R^2}{3\,\varepsilon_0}$ (b) $\frac{7\rho_0 R^2}{24\varepsilon_0}$

(c) $\frac{15\rho_0 R^2}{7\varepsilon_0}$ (d) $\frac{14\rho_0 R^2}{3\varepsilon_0}$

52. Three long, straight, wires in the x-z plane each carrying current I, cross at the origin of coordinates, as shown in figure. The magnitude of the magnetic field B is a function of x with y = z = 0 , is



(a) $\frac{3\mu_0 I}{2\pi x}$ (b) $\frac{3\mu_0 I}{2\pi x}$

(c) $\frac{\mu_0 I}{2\pi x}\left(1+2\sqrt{2}\right)$ (d) $\frac{\mu_0 I}{2\pi x}$

53. Suppose a square of side a lying in the xy plance with its centre at the origin carries current I in anticlockwise direction. The magnetic field on the x-axis at a distance d from the origin is given by (Assume d>>a)

(a) $-\frac{\mu_0 I}{4\pi d^2}\hat{x}$ (b) $-\frac{\mu_0 I a^2}{4\pi d^3}\hat{z}$

(c) $\frac{\mu_0 I}{4\pi d^3}\hat{y}$ (d)Zero

54. Consider $\overline{A}$ and V are vector and scalar potential $\overline{A}(\mathrm{r}, t) = -\frac{qt\hat{r}}{4\pi\varepsilon_0 r^3}, V(r,t) = 0$ and a scalar function

$\lambda = -\frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}$. Due to Gauge transformation, the transformed potential are

(a) $V'(r,t) = 0,\ \overline{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(b) $V'(r,t) = \frac{q}{4\pi\varepsilon_0 r^2},\ \overline{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(c) $V'(r,t) = \frac{q}{4\pi\varepsilon_0 r^2}\frac{1}{r^2},\ \overline{A}'(r,t) = -\frac{q}{4\pi\varepsilon_0}\frac{t}{r^2}\hat{r}$

(d) $V'(r,t) = \frac{q}{4\pi\varepsilon_0 r^2},\ \overline{A}'(r,t) = \frac{q}{4\pi\varepsilon_0}\frac{t}{r^3}\hat{r}$

55. The electric field of a plane polarized e.m. wave is given by $\vec{E} = E_0(\hat{x} + e^{i\varphi}\hat{y})e^{i(kz-\omega t)}$. The intensity associated with the wave is

(a) $\frac{\varepsilon_0 E_0^2 c}{2}\sqrt{(1-\cos^2\phi)}$ (b) $\varepsilon_0 E_0^2 c$

(c) $\varepsilon_0 E_0^2 c\sin^2\phi$ (d) $\varepsilon_0 E_0^2 c\cos^2\phi$

56. A particle moves in the one-dimensional potential $V(x) = \alpha x^6$, where $\alpha > 0$ is a constant. If the total energy of the particle is E, its time period in a periodic motion is proportional to

(a) $E^{-1/3}$ (b) $E^{-1/2}$

(c) $E^{1/3}$ (d) $E^{1/2}$

57. Hamitonial of a system is $H = \frac{(p_x - \propto x)^2}{2m} + \frac{1}{2}m\omega^2 x^2$. If at $t = 0,\ x = A,\ \dot{x} = v_0$ then $x(t)$ is given

(a) $A\cos\omega t+\frac{v_0}{\omega}\sin\omega t$ (b) $A\cos\omega t+\frac{v_0}{\omega}(t-\cos\omega t)$

(c) $A\cos\omega t+\frac{v_0}{\omega}(t+\cos\omega t)$ (d) $A(1-\sin\omega t)+\frac{v_0}{\omega}\sin\omega t$

58. Hamiltonian of a system is $H=\frac{p_x^2}{2m}e^{-\alpha t}+\frac{1}{2}kx^2e^{\alpha t}+\beta xp_x$ corresponding Lagrangian is :

(a) $\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2\right)e^{-\alpha t}-\beta x\dot{x}$ (b) $\left[\frac{1}{2}m(\dot{x}-\beta x)^2-\frac{1}{2}kx^2\right]e^{-\alpha t}$

(c) $\left[\frac{1}{2}m(\dot{x}-\beta x)^2-\frac{1}{2}kx^2\right]e^{\alpha t}$ (d) $\frac{1}{2}m\dot{x}^2e^{-\alpha t}-\frac{1}{2}kx^2e^{\alpha t}-\beta x\dot{x}$

59. The kinetic energy and potential energy for a double pendulum are given respectively by :

$$K.E.=ml^2\dot{\theta}_1^2+ml^2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2+\frac{1}{2}ml^2\dot{\theta}_2^2$$

$$P.E.=-2mgl\cos\theta_1-mgl\cos\theta_2$$

The K.e. and P.E. matrices for small oscillation of the double pendulum are respectively.

(a) $T=\begin{bmatrix}ml^2 & ml^2\\ ml^2 & \frac{1}{2}ml^2\end{bmatrix};V=\begin{bmatrix}mgl & 0\\ 0 & \frac{1}{2}mgl\end{bmatrix}$

(b) $T=\begin{bmatrix}2ml^2 & \frac{1}{2}ml^2\\ \frac{1}{2}ml^2 & ml^2\end{bmatrix};\mathrm{V}=\begin{bmatrix}2mgl & 0\\ 0 & mgl\end{bmatrix}$

(c) $T=\begin{bmatrix}2ml^2 & \frac{1}{2}ml^2\\ \frac{1}{2}ml^2 & ml^2\end{bmatrix};\mathrm{V}=\begin{bmatrix}mgl & 0\\ 0 & \frac{1}{2}mgl\end{bmatrix}$

(d) $T=\begin{bmatrix}2ml^2 & ml^2\\ ml^2 & ml^2\end{bmatrix};V=\begin{bmatrix}2mgl & 0\\ 0 & mgl\end{bmatrix}$

60. The Lagrangian of a system is given by $L=\frac{1}{2}(M+m)\dot{x}^2+ml\cos\theta\dot{x}\dot{\theta}+\frac{1}{2}ml^2\dot{\theta}^2-\frac{1}{2}Kx^2+mgl\cos\theta$ where x and θ are the generalized co-ordinate. The equation of motion are

(a) $M\ddot{X}+ml\cos\theta\ddot{\theta}-ml\sin\theta=-kx,\ \cos\theta\ddot{x}+l\ddot{\theta}=-g\sin\theta$

(b) $(M+m)\ddot{x}+ml\cos\theta\ddot{\theta}-ml\sin\theta\dot{\theta}^2=-kx,\ \sin\theta\ddot{x}+l\ddot{\theta}=-g\cos\theta$

(c) $M\ddot{x}+ml\sin\theta\ddot{\theta}-ml\cos\theta\dot{\theta}^2=-kx,\ \cos\theta\ddot{x}+l\ddot{\theta}=-g\sin\theta$

(d) $(M+m)\ddot{x}+ml\cos\theta\ddot{\theta}-ml\sin\theta\dot{\theta}^2=-kx,\ \cos\theta\ddot{x}+l\ddot{\theta}=-g\sin\theta$

61. Generating function for a canonical transformation is $F_1 = -\frac{q^2}{P}$, The other generating function $F_2(p,P)$ for this canonical transformation is

(a) $\frac{p^2P}{2}$ (b) $\frac{-p^2P}{2}$

(c) $\frac{-p^2P}{4}$ (d) $\frac{-p^2P}{4}$

62. The Hamiltonian of a one-dimensional system is $H = \frac{xp^2}{2m} + \frac{1}{2}kx$, where *m* and *k* are positive constants. The corresponding euler-Lagrange equation for the system is

(a) $m\ddot{x} + k = 0$ (b) $m\ddot{x} + 2\dot{x} + kx^2 = 0$

(c) $2mx\ddot{x} - m\dot{x}^2 + kx^2 = 0$ (d) $mx\ddot{x} + 2m\dot{x}^2 + kx^2 = 0$

63. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the *z* axis. Their velocities are given by

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z};\ \vec{v}_2 = (-0.8c)\hat{z}$

64. The Lagrangian of a system with one degree of freedom *q* is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to *q* then which one of the following statements is CORRECT?

(a) $p_q = 2\beta q$ and it is a conserved quantity.

(b) $p_q = 2\beta q$ and it is not a conserved quantity.

(c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.

(d) $q_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

65. A particle of mass *m* and coordinate *q* has the Lagrangian $L = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$ where λ is a constant. The Hamiltonian for the system is given by

(a) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2m^2}$ (b) $\frac{p^2}{2(m-\lambda q)}$

(c) $\frac{p^2}{2m} + \frac{\lambda q p^2}{2(m-\lambda q)^2}$ (d) $\frac{p\dot{q}}{2}$

66. The lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$$

Where *m* and *k* are positive constants. The frequencies of its normal modes are

(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{2m}(13 \pm \sqrt{73})}$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$

67. Lagrangian of a particle moving in *x-y plane is* $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m\omega^2(x^2 + y^2)$.

Which of the following path is not possible

(a) straight line (b) circle

(c) ellipse (d) parabola

68. The Hamiltonian of a system with *n* degrees of freedom is given by $H(q_1....q_n;\, p_1.....p_n;\, t)$, with an explicit dependence on the time *t*. Which of the following is correct?

(a) Different phase trajectories cannot intersect each other.

(b) H always represents the total energy of the system and is a constant of the motion.

(c) The equation $\dot{q}_i = \partial H / \partial p_i;\ \dot{p}_i = -\partial H / \partial_i$ are not valid since H has explicit time dependence.

(d) Any initial volume element in phase space remains unchanged in magnitude under time evolution.

69. A particle moves in one dimensional potential $V(x) = V_0[\delta(x+a) + \delta(x-a)]$ where V_0 is positive constant shape of phase space trajectory of the particle is

(a) parabola (b) rectangle

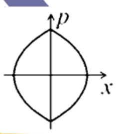
(c) circle (d) ellipse

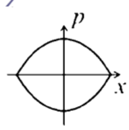
70. For canonical transformation $q, p \rightarrow Q, P$ such that $p = \sin Q,\ q = -P\operatorname{cosec}Q$ generating function $F(q, Q)$ is

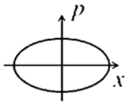
(a) $q\cos Q$ (b) $q\sin Q$

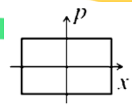
(c) $-q\sin Q$ (d) $-q\cos Q$

71. A particle moves in one dimension under a potential V(x) = $\alpha|x|$ with some non-zero total energy. Which one of the following best describes the particle trajectory in the phase space?

(a) 

(c) 

(c) 

(d) 

72. The energy of a one-dimensional system if force is given as $F(x) = -knx^{2n-1}$, where *k* and *n* are two positive constants, is E_0. The time period of oscillation τ satisfies (it is given as potential, V = 0 as *x* = 0)

(a) $\tau \propto E_0^{-\frac{1}{n}}$ (b) $\tau \propto E_0^{\frac{1-n}{2n}}$

(c) $\tau \propto E_0^{\frac{n-2}{2n}}$ (d) $\tau \propto E_0^{\frac{1+n}{2n}}$

73. The Largrangian of a system is given by $L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta,$ where $m \cdot \ell$ and *g* are constants

Which of the following is conserved?

(a) $\dot{\varphi}\sin^2\theta$ (b) $\dot{\varphi}\sin\theta$

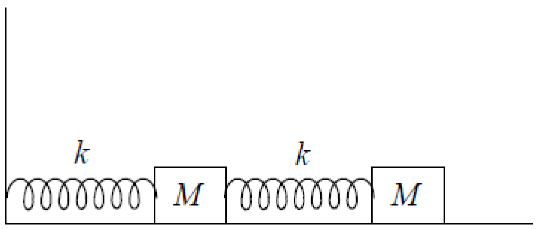
(c) $\frac{\dot{\varphi}}{\sin\theta}$ (d) $\frac{\dot{\varphi}}{\sin^2\theta}$

74. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

(a) $V = -2T;\ E = -T$ (b) $V = -T;\ E = 0$

(c) $V = -\frac{T}{2};\ E = \frac{T}{2}$ (d) $V = \frac{-3T}{2};\ E = \frac{-T}{2}$

75. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is *k*. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is :



(a) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$

(c) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$ (d) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$